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Short Communication

Highly accurate free vibration eigenvalues for the completely free orthotropic plate

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1. Introduction

In the history of free vibration analysis of rectangular plates with classical edge conditions, the practice has been to leave problems involving plates with free edges till the last. This is no doubt due to the difficulties encountered in trying to satisfy the free edge conditions. Because of the mixed derivatives involved in formulating these conditions, it has been difficult, for example, to choose suitable functions to represent plate lateral deflection when attempting to solve these problems by means of the Rayleigh–Ritz method.

There is a particular need for access to tabulations of highly accurate eigenvalues for the completely free orthotropic plate. This was recognized by Moussu and Nivoit [1], for example, in their attepmts to infer elastic constants of orthotropic plates by experimenting with free vibration of these same plates in a completely free condition. There is also a need for accurate eigenvalue listings against which other analysts can compare their results.

Accurate eigenvalues for completely free orthotropic plates presented here have been computed by the superposition method. Fortunately, with this method no functions need be chosen to represent plate lateral displacement. The governing differential equation is satisfied exactly

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Nomenclature

a, b quarter-plate edge dimensions
D_x, D_y $E_x h^3/12(1-v_x v_y)$, and $E_y\ h^3/12(1-v_x v_y)$, respectively
DHX, DHY H/D_x and H/D_y, respectively
D_t $G_{xy}\ h^3/12$
E_x, E_y modulus of elasticity of plate material associated with x and y directions respectively
G_{xy} modulus of elasticity in shear related to x–y plane of plate
h plate thickness
H $2H = v_y D_x + v_x D_y + 4D_t$
x,y coordinates measured along edges of quarter plate
v_x, v_y the Poisson ratios associated with x and y directions, respectively along the plate; the product $v_x\ v_y$ is taken as equal to 0.333^2 in all work reported here
ξ, η x/a, and y/b, respectively
φ plate aspect ratio, b/a
$\varphi^{|}$ inverse of plate aspect ratio
λ^2 eigenvalue, $=\omega a^2\sqrt{\rho/D_x}$
λ^{*2} alternate formulation of eigenvalue $=\omega b^2\sqrt{\rho/D_x}$
ω circular frequency of plate vibration
ρ mass of plate per unit area

throughout the domain of the plate, and boundary conditions are satisfied to any desired degree of exactitude. These desirable characteristics of the method are recognized in the above reference by Moussu and Nivoit.

In an earlier publication [2], the author obtained accurate solutions for a limited class of completely free orthotropic plate vibration problems. Findings presented here are the result of a comprehensive study conducted without restriction in regard to plate geometry or interrelationships between the basic plate elastic properties.

2. Mathematical procedure

The mathematical procedure followed here to generate the tabulated eigenvalues is virtually identical to that described in an earlier paper related to the free vibration of completely free orthotropic plates resting on symmetrically distributed point supports [3]. The only difference is that, since there are no point supports acting on the plates under study here, the contribution of these supports toward dynamic equilibrium of the earlier analysis is neglected. This is taken care of by simply deleting the last row and column of the eigenvalue matrices related to the earlier problems. There is therefore no need to describe the analytical procedure again here. A fairly comprehensive discussion of orthotropic plate properties as they ulate to the work presented here is to be found in Ref. [4].

We again take advantage of symmetry and only one quarter of the plate is analyzed. The quarter plate segment is illustrated in Fig. 1. It will be appreciated that all plate free vibration modes will either be fully symmetric or fully anti-symmetric about the main plate central-axis, or symmetric about one axis and anti-symmetric about the other. This gives rise to three distinct families of free vibration modes, each family being analyzed separately. Selection of the

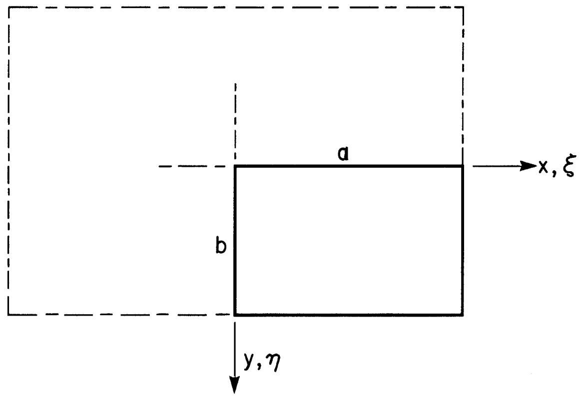

Fig. 1. Completely free orthotropic plate indicated by broken lines. Quarter-plate analyzed indicated by solid lines.

pairs of building blocks (forced vibration solutions), their superposition, and generation of their associated eigenvalue matrices is described in detail in Ref. [3], and will not be repeated here.

3. Presentation of computed results

Since in the present analysis there are no concentrated forces and no use of Dirac functions, convergence is much more rapid. It is found that use of 15 terms in the building block solutions virtually guarantees that computed eigenvalues will have four significant digit accuracy. In fact, 15 terms have been utilized in all computations carried out in connection with the present work and that is why eigenvalues are listed to four significant digits.

Eigenvalues are tabulated in Table 1 for the first four fully symmetric modes of the completely free orthotropic plate. Plate aspect ratios are allowed to take on six different values ranging from 1.0 to 3.0. The orthotropic parameters, DHY and DHX (see list of symbols), are each allowed to take on values of 1.0, 1.5, and 2.0, as well as $\frac{1}{1.5}$, and $\frac{1}{2.0}$. This gives rise to 24 sub-tables for the fully symmetric mode family. A corresponding set of results for the fully anti-symmetric mode family is presented in Table 2. Two sets of corresponding results are presented in Tables 3 and 4 for the symmetric–anti-symmetric mode family. The first set has a range of plate aspect ratios identical to that used in Tables 1 and 2. In the second set, the plate aspect ratio is replaced by its inverse. This is necessary in order that data provided for this latter family of modes will be complete. It is pointed out that all modes related to the data of Tables 3 and 4 are symmetric with respect to the ξ-axis and anti-symmetric with respect to the η-axis of Fig. 1.

In Table 5 eigenvalues are tabulated for the above three mode families of isotropic plates. These eigenvalues may be required in order to permit interpolation.

Table 1
Eigenvalues, λ^2, for symmetric–symmetric free vibration modes

	φ (DHY = 1.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.123	4.040	2.873	1.627	1.042	0.7227
2	7.026	5.701	5.563	5.506	5.335	3.963
3	19.21	15.38	12.99	9.039	5.978	5.573
4	29.82	23.39	16.29	10.41	8.864	7.941
	φ (DHY = 1.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.213	4.529	3.302	1.879	1.204	0.8354
2	7.977	5.879	5.597	5.520	5.458	4.562
3	22.17	17.67	14.84	10.45	6.771	5.610
4	29.89	26.95	18.80	11.70	9.827	8.695
	φ (DHY = 1.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.178	2.757	1.923	1.083	0.6923	0.4803
2	5.723	5.546	5.510	5.369	3.779	2.628
3	12.82	10.52	9.101	6.116	5.560	5.511
4	24.25	15.57	10.89	7.786	6.980	6.361
	φ (DHY = 1.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.678	2.391	1.665	0.9365	0.5989	0.4155
2	5.619	5.523	5.493	4.988	3.268	2.269
3	11.13	9.269	8.112	5.681	5.540	5.377
4	20.99	13.47	9.458	7.159	6.544	5.774
	φ (DHY = 1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.183	2.761	1.927	1.085	6.938	0.4813
2	5.737	5.557	5.520	5.390	3.806	2.645
3	15.69	12.75	10.54	6.164	5.567	5.522
4	24.35	15.64	11.30	8.983	7.850	6.594
	φ (DHY = 1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.809	3.359	2.357	1.330	0.8509	0.5904
2	6.118	5.604	5.540	5.484	4.643	3.250
3	19.09	15.38	12.82	7.458	5.619	5.552
4	29.24	19.15	13.55	10.43	8.928	7.861

Table 1 (*continued*)

	φ (DHY = 1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.065	3.839	2.716	1.536	0.9834	0.6824
2	6.712	5.667	5.559	5.505	5.215	3.757
3	21.95	17.60	14.71	8.609	5.796	5.568
4	29.86	22.11	15.52	11.70	9.885	8.707
	φ (DHY = 1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.487	2.260	1.573	0.8847	0.5655	0.3923
2	5.609	5.529	5.499	4.787	3.101	2.151
3	12.92	10.64	8.635	5.636	5.542	5.243
4	19.88	12.79	9.551	7.872	7.017	5.669
	φ (DHY = 1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.038	1.959	1.362	0.7653	0.4892	0.3394
2	5.565	5.513	5.477	4.171	2.679	1.857
3	11.30	9.402	7.502	5.578	5.518	4.597
4	17.21	11.09	8.567	7.257	6.472	5.583
	φ (DHY = 2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.686	2.399	1.671	0.9401	0.6010	0.4169
2	5.641	5.541	5.510	5.053	3.307	2.294
3	15.68	12.69	9.322	5.704	5.553	5.434
4	21.14	13.67	11.14	9.031	7.819	5.849
	φ (DHY = 2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.386	2.927	2.045	1.153	0.7372	0.5114
2	5.813	5.572	5.529	5.442	4.053	2.819
3	19.01	15.32	11.45	6.517	5.579	5.537
4	25.86	16.66	13.19	10.47	8.951	7.052
	φ (DHY = 2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.811	3.361	2.359	1.332	0.8520	0.5911
2	6.124	5.608	5.543	5.488	4.659	3.262
3	21.81	17.53	13.24	7.486	5.622	5.554
4	29.30	19.21	14.95	11.73	9.913	8.127

Table 1 (*continued*)

	φ (DHY = 2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.041	1.961	1.363	0.7665	0.4899	0.3399
2	5.573	5.520	5.484	4.193	2.691	1.865
3	12.97	10.52	7.605	5.582	5.525	4.624
4	17.25	11.30	9.506	7.925	6.674	5.586
	φ (DHY = 2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.643	1.698	1.180	0.6630	0.4238	0.2940
2	5.546	5.502	5.442	3.632	2.324	4.611
3	11.38	9.188	6.612	5.559	5.446	4.004
4	14.94	9.958	8.576	7.308	5.889	5.563
	φ (DHY = 1/1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.117	4.036	2.869	1.624	1.039	0.7212
2	7.009	5.687	5.552	5.496	5.309	3.934
3	15.70	12.58	10.70	8.453	5.941	5.566
4	29.70	23.29	16.22	9.342	7.697	7.042
	φ (DHY = 1/1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.230	4.701	3.487	1.989	1.275	0.8847
2	8.413	5.993	5.609	5.518	5.467	4.787
3	19.39	15.37	12.92	10.05	7.103	5.636
4	29.82	28.32	19.88	11.31	8.798	7.872
	φ (DHY = 1/1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.268	5.007	3.977	2.295	1.473	1.023
2	9.653	6.504	5.686	5.533	5.495	5.297
3	22.47	17.74	14.82	11.41	8.191	5.904
4	29.89	29.73	22.96	13.01	9.782	8.628
	φ (DHY = 1/1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.803	3.3494	2.348	1.324	0.8472	0.5879
2	6.082	5.579	5.519	5.460	4.571	3.206
3	12.69	10.33	8.950	7.119	5.603	5.535
4	28.84	18.99	13.22	7.866	6.881	6.436

Table 1 (*continued*)

	φ (DHY = 1/1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.376	2.914	2.034	1.146	0.7329	0.5084
2	5.766	5.537	5.498	5.392	3.972	2.769
3	10.91	9.031	7.957	6.304	5.557	5.505
4	25.49	16.41	11.43	7.107	6.446	6.104
	φ (DHY = 1/2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.202	4.522	3.293	1.872	1.199	0.8323
2	7.946	5.851	5.577	5.503	5.434	4.502
3	15.74	12.48	10.57	8.470	6.624	5.598
4	29.68	26.71	18.67	10.57	7.664	6.973
	φ (DHY = 1/2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.262	5.004	3.974	2.292	1.470	1.021
2	9.639	6.492	5.676	5.526	5.488	5.277
3	19.57	15.37	12.85	10.02	8.017	5.882
4	29.80	29.63	22.90	12.93	8.859	7.810
	φ (DHY = 1/2, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.285	5.154	4.468	2.648	1.699	1.180
2	11.09	7.286	5.838	5.546	5.506	5.442
3	22.76	17.81	14.80	11.38	9.188	6.612
4	29.88	29.79	26.40	14.94	9.958	8.576
	φ (DHY = 1/2, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.053	3.826	2.703	1.527	0.9776	0.6785
2	6.658	5.627	5.526	5.473	5.113	3.684
3	12.60	10.16	8.776	7.281	5.739	5.545
4	29.43	21.83	15.20	8.657	6.801	6.369
	φ (DHY = 1/2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.800	3.343	2.343	1.322	0.8457	0.5871
2	6.048	5.555	5.497	5.434	4.531	3.185
3	10.74	8.811	7.760	6.621	5.588	5.514
4	28.46	18.85	13.12	7.541	6.349	6.055

Table 2
Eigenvalues, λ^2, for anti-symmetric–anti-symmetric free vibration modes

	φ (DHY = 1.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.173	3.338	2.775	2.068	1.645	1.364
2	19.35	15.34	11.72	7.822	5.846	4.665
3	22.89	18.52	17.44	16.31	12.66	9.614
4	45.68	36.58	29.97	18.49	16.12	15.71
	φ (DHY = 1.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.903	3.925	3.266	2.438	1.941	1.611
2	2.101	17.66	13.64	9.137	6.841	5.467
3	26.38	19.82	18.26	16.89	14.68	11.21
4	52.26	41.64	34.58	21.36	16.52	16.04
	φ (DHY = 1.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.544	2.037	1.689	1.254	0.9952	0.8242
2	14.18	9.944	7.493	4.935	3.654	2.896
3	17.37	16.42	15.92	11.75	8.143	6.124
4	31.46	25.86	19.76	15.75	14.77	10.93
	φ (DHY = 1.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.102	1.675	1.388	1.030	0.8167	0.6761
2	12.20	8.452	6.335	4.141	3.050	2.408
3	16.61	15.97	15.40	10.03	6.905	5.168
4	27.76	24.33	17.18	15.51	12.80	9.315
	φ (DHY = 1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.407	2.718	2.256	1.679	1.336	1.109
2	15.80	11.37	8.782	6.000	4.551	3.667
3	18.69	17.35	16.64	13.07	9.329	7.195
4	37.30	28.97	21.22	16.17	15.60	12.15
	φ (DHY = 1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.275	3.416	2.839	2.118	1.687	1.401
2	19.30	14.09	10.91	7.478	5.686	4.591
3	20.92	18.52	17.51	15.89	11.57	8.948
4	44.84	35.57	26.14	16.98	16.13	14.95

Table 2 (*continued*)

	φ (DHY = 1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.995	3.994	3.323	2.483	1.980	1.645
2	21.06	16.35	12.70	8.712	6.631	5.359
3	23.62	19.64	18.32	16.91	13.44	10.41
4	51.22	40.89	30.30	19.06	16.48	16.03

	φ (DHY = 1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.684	3.138	1.772	1.317	1.047	0.8681
2	12.80	9.127	7.022	4.773	3.608	2.899
3	17.37	16.51	15.82	10.53	7.474	5.744
4	31.21	23.51	17.40	15.76	13.14	9.772

	φ (DHY = 1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.244	1.786	1.479	1.098	0.8721	0.7231
2	10.96	7.771	5.959	4.031	3.036	2.434
3	16.70	16.04	14.35	8.988	6.354	4.868
4	27.64	20.27	16.13	15.44	11.26	8.338

	φ (DHY = 2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.467	2.765	2.296	1.711	1.362	1.132
2	14.86	10.81	8.450	5.872	4.501	3.652
3	18.65	17.40	16.66	12.22	8.864	6.925
4	36.95	26.26	19.43	16.17	14.92	11.34

	φ (DHY = 2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.326	3.455	2.873	2.145	1.711	1.422
2	18.24	13.38	10.48	7.294	5.602	4.553
3	20.46	18.56	17.56	15.05	10.98	8.590
4	44.36	32.35	23.89	16.75	16.13	14.00

	φ (DHY = 2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.039	4.029	3.353	2.508	2.002	1.666
2	20.78	15.54	12.18	8.482	6.518	5.302
3	22.41	19.65	18.37	16.83	12.74	9.981
4	50.61	37.47	27.66	17.89	16.49	15.90

Table 2 (*continued*)

	φ (DHY = 2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.752	2.191	1.817	1.352	1.076	0.8931
2	12.00	8.690	6.777	4.692	3.587	2.904
3	17.42	16.56	15.24	9.850	7.118	5.548
4	30.79	21.31	16.57	15.74	12.13	9.137

	φ (DHY = 2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.319	1.844	1.528	1.135	0.9030	0.7495
2	10.27	7.413	5.767	3.979	3.034	2.452
3	16.77	16.04	13.24	8.420	6.065	4.715
4	26.84	18.43	16.04	14.86	10.39	7.807

	φ (DHY = 1/1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.128	2.502	2.080	1.547	1.228	1.016
2	17.37	13.74	10.27	6.599	4.797	3.753
3	21.28	17.24	16.48	15.56	11.36	8.414
4	38.52	30.87	26.34	17.23	15.71	15.11

	φ (DHY = 1/1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.026	3.226	2.684	2.001	1.590	1.317
2	19.20	16.66	12.80	8.301	6.074	4.773
3	20.05	18.89	17.37	16.31	14.07	10.53
4	46.82	37.05	31.21	21.03	16.17	15.76

	φ (DHY = 1/1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.767	3.822	3.184	2.376	1.890	1.567
2	20.79	18.46	14.88	9.724	7.134	5.620
3	30.17	21.04	18.24	16.83	15.83	12.28
4	53.32	42.32	35.39	24.40	17.09	16.03

	φ (DHY = 1/1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.370	1.893	1.571	1.168	0.9259	0.7659
2	15.50	11.06	8.163	5.180	3.734	2.903
3	17.89	16.30	15.85	13.32	9.063	6.664
4	31.80	25.98	22.23	15.77	15.36	12.45

Table 2 (*continued*)

	φ (DHY = 1/1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	1.898	1.515	1.258	0.9341	0.7405	0.6123
2	13.71	9.396	6.883	4.319	3.086	2.383
3	16.61	15.82	15.48	11.41	7.673	5.606
4	27.88	23.19	19.62	15.48	14.56	10.61
	φ (DHY = 1/2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.973	2.381	1.981	1.475	1.171	0.9687
2	17.25	14.79	11.07	9.643	4.947	3.812
3	23.49	17.38	16.40	15.66	12.40	9.074
4	39.25	31.06	26.41	18.88	15.71	15.39
	φ (DHY = 1/2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.887	3.118	2.597	1.937	1.539	1.274
2	18.98	17.14	13.76	8.762	6.294	4.881
3	28.93	19.94	17.35	16.26	15.10	11.36
4	47.88	37.48	31.38	25.20	16.45	15.75
	φ (DHY = 1/2.0 DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.637	3.723	3.104	2.318	1.844	1.528
2	20.55	18.50	15.87	10.27	7.413	5.767
3	33.54	22.82	18.38	16.77	16.04	13.24
4	53.68	42.96	35.68	26.84	18.43	16.04
	φ (DHY = 1/2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	2.192	1.753	1.457	1.084	0.8602	0.7138
2	15.83	12.04	8.792	5.428	3.823	2.919
3	19.18	16.21	15.75	14.49	9.901	7.174
4	32.20	25.98	22.55	16.01	15.37	13.73
	φ (DHY = 1/2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	1.697	1.356	1.127	0.8390	0.6657	0.5504
2	14.65	10.26	7.405	4.507	3.136	2.371
3	17.00	15.68	15.38	12.60	8.379	6.026
4	28.08	23.10	20.39	15.46	15.07	11.76

Table 3
Eigenvalues, λ^2, for symmetric–anti-symmetric free vibration modes $\varphi \geq 10$

	φ (DHY = 1.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	10.66	7.975	6.358	4.526	3.514	2.872
2	15.30	15.23	15.19	12.28	8.854	6.878
3	30.65	25.58	19.64	15.34	15.26	12.92
4	40.17	27.40	23.30	20.03	17.38	15.36
	φ (DHY = 1.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	12.36	9.315	7.440	5.307	4.127	3.377
2	15.42	15.26	15.23	14.20	10.33	8.037
3	34.61	28.75	22.77	15.48	15.31	14.86
4	46.29	31.50	25.64	21.56	19.32	15.55
	φ (DHY = 1.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.796	5.019	3.972	2.799	2.159	1.756
2	15.18	15.05	12.57	7.889	5.619	4.329
3	22.44	17.51	15.36	15.16	11.26	8.319
4	26.57	20.38	18.79	16.55	15.31	13.90
	φ (DHY = 1.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.724	4.202	3.311	2.321	1.785	1.449
2	15.13	14.38	10.75	6.683	4.731	3.628
3	20.35	15.70	15.28	14.07	9.600	7.059
4	23.00	18.93	17.76	15.43	15.21	11.93
	φ (DHY = 1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	8.185	6.203	4.995	3.595	2.808	2.304
2	15.24	15.18	13.92	9.116	6.692	5.275
3	26.06	19.01	15.46	15.27	12.51	9.477
4	28.27	23.01	20.84	17.86	15.35	15.02
	φ (DHY = 1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	10.17	7.734	6.239	4.504	3.526	2.897
2	15.28	15.24	15.17	11.31	8.330	6.580
3	30.68	23.41	17.57	15.33	15.10	11.75
4	34.57	26.33	23.38	20.11	15.76	15.34

Table 3 (*continued*)

	φ (DHY = 1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.81	9.010	7.274	5.258	4.122	3.391
2	15.34	15.26	15.24	13.13	9.696	7.667
3	34.58	27.12	20.34	15.37	15.30	13.64
4	39.96	29.27	25.66	21.66	18.03	15.38
	φ (DHY = 1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.532	4.931	3.960	2.840	2.212	1.811
2	15.19	14.69	11.28	7.298	5.335	4.192
3	21.97	15.83	15.32	14.39	10.07	7.596
4	23.62	20.50	18.94	15.46	15.27	12.28
	φ (DHY = 1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.531	4.160	3.332	2.381	1.851	1.514
2	15.13	12.93	9.641	6.200	4.514	3.536
3	19.15	15.38	15.27	12.42	8.597	6.461
4	21.37	19.12	17.90	15.34	14.27	10.51
	φ (DHY = 2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.992	6.121	4.963	3.602	2.828	2.328
2	15.24	15.16	13.08	8.703	6.484	5.166
3	24.99	17.56	15.37	15.24	11.70	8.975
4	26.83	23.07	20.92	16.62	15.34	14.03
	φ (DHY = 2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.912	7.606	6.174	4.491	3.533	2.913
2	15.27	15.25	15.12	10.78	8.048	6.422
3	30.36	21.59	16.46	15.32	14.40	11.11
4	31.69	26.33	23.45	19.97	15.42	15.33
	φ (DHY = 2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.51	8.845	7.183	5.230	4.119	3.400
2	15.31	15.27	15.25	12.51	9.354	7.468
3	34.45	25.04	18.96	15.35	15.29	12.90
4	36.33	29.20	25.71	21.70	16.85	15.36

Table 3 (*continued*)

	φ (DHY = 2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.402	4.891	3.957	2.863	2.243	1.843
2	15.19	13.89	10.56	6.985	5.189	4.124
3	20.34	15.46	15.31	13.32	9.422	7.210
4	23.30	20.60	19.01	15.37	15.02	11.34
	φ (DHY = 2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.440	4.145	3.348	2.416	1.889	1.551
2	15.10	11.98	9.035	5.949	4.406	3.494
3	17.58	15.34	15.26	11.44	8.052	6.145
4	21.35	19.24	17.84	15.32	13.08	9.715
	φ (DHY = 1/1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.104	6.605	5.156	3.574	2.732	2.210
2	15.24	15.18	15.08	10.93	7.641	5.791
3	25.81	22.13	18.08	15.30	15.09	11.65
4	38.48	25.64	20.62	18.22	16.14	15.32
	φ (DHY = 1/1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.35	8.323	6.532	4.559	3.500	2.840
2	15.37	15.23	15.19	13.55	9.580	7.298
3	30.52	25.66	21.97	15.43	15.28	14.39
4	50.05	31.51	23.62	19.89	18.18	15.46
	φ (DHY = 1/1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	13.06	9.744	7.673	5.374	4.136	3.361
2	15.62	15.27	15.22	14.97	11.19	8.552
3	34.60	28.73	24.85	16.29	15.32	15.24
4	49.19	36.49	26.73	21.43	19.32	16.94
	φ (DHY = 1/1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.201	5.169	4.005	2.750	2.090	1.684
2	15.16	15.09	14.09	8.712	6.032	4.538
3	22.15	19.25	15.57	15.22	12.80	9.305
4	31.20	21.08	18.58	17.00	15.34	15.14

Table 3 (*continued*)

	φ (DHY = 1/1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.036	4.291	3.301	2.244	1.696	1.362
2	15.11	14.94	12.20	7.365	5.057	3.779
3	20.11	17.20	14.98	15.05	10.93	7.884
4	26.83	18.90	17.52	16.18	15.25	13.71
	φ (DHY = 1/2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.632	6.854	5.265	3.572	2.696	2.164
2	15.25	15.16	15.10	11.88	8.165	6.087
3	25.57	21.98	19.38	15.32	15.19	12.76
4	43.37	28.64	21.09	18.09	16.86	15.34
	φ (DHY = 1/2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.97	8.665	6.708	4.595	3.490	2.812
2	15.46	15.23	15.18	14.48	10.25	7.696
3	30.40	25.51	22.46	15.71	15.29	15.09
4	48.95	35.51	25.47	19.78	18.14	16.10
	φ (DHY = 1/2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	13.59	10.16	7.900	5.440	4.145	3.348
2	15.95	15.29	15.22	15.10	11.98	9.035
3	34.59	28.62	24.95	17.58	15.34	15.26
4	49.25	40.89	29.44	21.35	19.24	17.84
	φ (DHY = 1/2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.605	5.334	4.055	2.711	2.028	1.618
2	15.14	15.08	14.77	9.469	6.430	4.750
3	21.83	19.30	16.35	15.21	14.06	10.20
4	35.22	23.19	18.47	16.89	15.43	15.22
	φ (DHY = 1/2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.358	4.402	3.312	2.180	1.615	14.82
2	15.07	14.97	13.39	8.001	5.379	8.638
3	19.76	17.81	15.40	15.10	12.10	3.938
4	30.24	20.00	17.31	16.27	15.25	1.280

Table 4
Eigenvalues, λ^{*2}, for symmetric–anti-symmetric free vibration modes $\varphi^1 \geq 10$

	φ^1 (DHY = 1.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	10.66	8.926	6.555	3.708	2.369	1.642
2	15.30	10.28	8.905	7.942	7.276	5.364
3	30.65	23.78	19.61	12.61	8.107	7.371
4	40.18	31.57	21.99	15.06	12.54	10.92
	φ^1 (DHY = 1, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	12.36	9.391	6.604	3.715	2.372	1.643
2	15.42	11.39	10.29	9.187	7.672	5.384
3	34.61	27.10	21.78	12.36	8.917	8.488
4	46.29	31.64	22.74	17.39	14.57	11.39
	φ (DHY = 1.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.796	6.005	5.447	3.657	2.357	1.636
2	15.18	9.759	6.896	5.247	4.934	4.710
3	22.44	16.77	13.49	9.967	7.745	5.508
4	26.57	25.58	21.86	12.32	8.348	7.217
	φ^1 (DHY = 1.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.724	5.097	4.690	3.595	2.348	1.633
2	15.13	9.722	6.803	4.562	4.252	4.100
3	20.35	14.99	11.89	8.648	7.014	5.417
4	23.00	22.06	21.16	12.28	7.906	6.216
	φ^1 (DHY = 1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	8.185	7.011	6.090	3.702	2.373	1.646
2	15.24	9.804	7.053	5.706	5.260	4.922
3	26.06	19.94	16.21	11.93	7.880	5.575
4	28.27	26.67	21.91	12.47	9.862	8.494
	φ^1 (DHY = 1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	10.17	8.566	6.557	3.723	2.379	1.648
2	15.28	9.981	8.131	7.017	6.443	5.346
3	30.68	23.73	19.49	12.33	7.939	6.323
4	34.57	31.36	21.99	14.73	12.09	10.43

Table 4 (*continued*)

	φ^1 (DHY = 1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.81	9.346	6.628	3.731	2.381	1.649
2	15.34	10.65	9.358	8.124	7.317	5.393
3	34.58	26.97	21.84	12.36	8.106	7.260
4	39.96	31.64	22.46	16.95	13.97	11.39
	φ^1 (DHY = 1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.532	5.633	5.042	3.645	2.364	1.642
2	15.19	9.753	6.829	4.669	4.274	4.063
3	21.97	16.93	13.57	9.899	7.742	5.466
4	23.62	21.70	20.87	12.33	8.159	6.905
	φ^1 (DHY = 1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.531	4.791	4.320	3.509	2.351	1.638
2	15.13	9.728	6.785	4.148	3.688	3.511
3	19.15	15.19	12.03	8.642	6.902	5.413
4	21.37	18.73	18.16	12.31	7.904	5.983
	φ^1 (DHY = 2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.992	6.765	5.893	3.709	2.380	1.650
2	15.24	9.788	6.927	5.294	4.802	4.496
3	24.99	19.97	16.19	11.89	7.888	5.513
4	26.83	23.91	21.80	12.42	9.692	8.273
	φ^1 (DHY = 2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.912	8.312	6.554	3.733	2.386	1.653
2	15.27	9.887	7.722	6.503	5.898	5.257
3	30.36	23.70	19.41	12.35	7.922	5.808
4	31.69	29.26	21.99	14.54	11.84	10.15
	φ^1 (DHY = 2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.51	9.283	6.646	3.742	2.389	1.654
2	15.31	10.30	8.853	7.530	6.790	5.387
3	34.45	26.88	21.86	12.37	7.977	6.566
4	36.33	31.56	22.30	16.71	13.67	11.38

Table 4 (*continued*)

	φ^1 (DHY = 2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.402	5.441	4.817	3.625	2.368	1.646
2	15.19	9.752	6.807	4.366	3.903	3.671
3	20.34	16.98	13.61	9.869	7.739	5.467
4	23.30	19.48	18.70	12.34	8.070	6.747
	φ^1 (DHY = 2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	5.440	4.638	4.125	3.399	2.352	1.642
2	15.10	9.732	6.779	3.987	3.373	3.169
3	17.58	15.21	12.10	8.646	6.852	5.420
4	21.35	16.88	16.16	12.31	7.905	5.868
	φ^1 (DHY = 1/1.5, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.104	8.063	6.425	3.682	2.356	1.633
2	15.24	9.982	8.165	7.435	6.997	5.318
3	25.81	19.77	16.26	12.03	7.984	7.123
4	28.48	31.42	21.88	12.81	10.68	9.532
	φ^1 (DHY = 1/1.5, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.35	9.174	6.548	3.695	2.360	1.635
2	15.37	10.96	9.967	9.142	7.633	5.353
3	30.52	23.82	19.78	12.30	9.042	8.705
4	50.05	31.56	21.99	15.54	13.20	11.25
	φ^1 (DHY = 1/1.5, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	13.06	9.408	6.583	3.701	2.362	1.637
2	15.62	12.48	11.54	10.52	7.713	5.366
3	34.60	27.26	21.72	12.43	10.38	10.00
4	49.19	31.63	23.12	18.00	15.27	11.44
	φ^1 (DHY = 1/1.5, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.201	6.535	5.910	3.658	2.349	1.630
2	15.16	9.777	7.089	6.028	5.771	5.188
3	22.15	16.54	13.39	10.10	7.764	5.909
4	31.20	30.02	21.78	12.30	8.655	7.682

Table 4 (*continued*)

	φ^1 (DHY = 1/1.5, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.036	5.544	5.186	3.626	2.342	1.627
2	15.11	9.712	6.850	5.195	4.978	4.781
3	20.11	14.70	11.72	8.699	7.207	5.494
4	26.83	26.19	21.66	12.23	7.924	6.575
	φ^1 (DHY = 1/2.0, DHX = 1.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	9.632	8.520	6.460	3.674	2.350	1.629
2	15.25	10.20	8.896	8.296	7.457	5.314
3	25.57	19.68	16.32	12.06	8.432	8.060
4	43.37	31.40	21.84	13.07	11.16	10.11
	φ^1 (DHY = 1/2.0, DHX = 1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	11.97	9.266	6.541	3.685	2.354	1.631
2	15.46	11.73	10.93	10.16	7.665	5.339
3	30.40	23.87	19.94	12.34	10.13	9.827
4	48.95	31.52	22.01	16.01	13.83	11.34
	φ^1 (DHY = 1/2.0, DHX = 2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	13.59	9.410	6.568	3.691	2.356	1.632
2	15.95	13.49	12.68	11.44	7.706	5.350
3	34.59	27.41	21.68	12.74	11.68	10.93
4	49.25	31.62	23.47	18.59	15.85	11.85
	φ^1 (DHY = 1/2.0, DHX = 1/1.5)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	7.605	7.024	6.175	3.655	2.344	1.626
2	15.14	9.805	7.426	6.721	6.465	5.255
3	21.83	16.35	13.34	10.26	7.813	6.598
4	35.22	31.08	21.73	12.28	8.979	8.138
	φ^1 (DHY = 1/2.0, DHX = 1/2.0)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	6.358	5.969	5.597	3.633	2.338	1.624
2	15.07	9.702	6.937	5.775	5.597	5.116
3	19.76	14.45	11.59	8.791	7.407	5.808
4	30.24	29.34	21.61	12.19	7.969	6.938

Table 5
Free vibration eigenvalues of completely free isotropic plate

	φ (Sym–sym mode eigenvalues, λ^2)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	4.806	3.355	2.353	1.128	0.8492	0.5892
2	6.106	5.595	5.533	5.476	4.613	3.230
3	15.69	12.69	10.82	7.382	5.613	5.547
4	29.11	19.09	13.31	8.924	7.785	7.093
	φ (Anti-sym–anti-sym mode eigenvalues, λ^2)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	3.292	2.629	2.182	1.623	1.289	1.069
2	17.03	12.41	9.401	6.245	4.650	3.699
3	19.24	17.27	16.57	14.51	10.19	7.705
4	37.81	30.65	24.35	16.26	15.72	13.58
	φ (Sym–anti-sym mode eigenvalues, λ^2, $\varphi \geq 1.0$)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	8.558	6.362	5.057	3.584	2.775	2.263
2	15.23	15.18	14.88	9.885	7.085	5.485
3	26.05	21.36	16.16	15.29	13.95	10.41
4	32.68	23.16	20.70	18.33	17.40	15.29
	φ^1 (Sym–anti-sym mode eigenvalues, λ^{*2}, $\varphi^1 \geq 1.0$)					
Mode	1.0	1.25	1.50	2.0	2.5	3.0
1	8.558	7.468	6.312	3.692	2.364	1.639
2	15.23	9.854	7.438	6.453	6.060	5.275
3	26.05	19.87	16.23	11.98	7.882	6.078
4	32.68	30.82	21.91	12.58	10.19	8.920

4. Conclusions

Computed results presented here constitute the most comprehensive compilation of highly accurate eigenvalues for the completely free orthotropic plate to appear in the literature. They will not only prove valuable for design purposes but will also prove useful in inverse problems where experimentally measured frequencies are utilized to infer material orthotropic properties.

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